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ABSTRACT

This paper develops several theoretical implications of a lexicographic preference ordering and then examines the relevance of these results for resource allocation analyses in elementary and secondary education. Because of this unique ordering, no substitution effects are possible, and the income effects are constrained and influence the demand for only one good or factor. It appears possible to observe a relative decrease in the demand for the "favored" good or factor, even though both goods are normal or both factors have positive marginal products. If this model accurately reflects the preference ordering for local public school administrators, a standard economic analysis of the current crises in the teacher labor market could yield inappropriate conclusions regarding the adjustment process on the demand side. (Author/DN)

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THE INFLUENCE OF LEXICOGRAPHIC PREFERENCE ORDERING ON RELATIVE DEMANDS UNDER ALTERNATIVE PRICE RATIOS--A GRAPHICAL PRESENTATION

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In his *Theory of Value*,* Debreu cites lexicographic ordering as an example of a completely preordered preference set which cannot be represented by a real-valued function. This paper develops several theoretical implications of this type of preference ordering and then examines the relevance of these results for resource allocation analyses in elementary and secondary education. With two normal goods (or factors of production) and standard assumptions regarding consumer preferences (or production functions), a change in the price ratio would ordinarily lead to a change in the relative demands for the "favored" good if consumers maximize their utility (output). Altering the assumed preference ordering by introducing lexicographic ordering undermines this general prediction. In the normal case, the substitution effect combines with the income effect for the favored good and the change in relative demands is predicated on the assumption that the combined income and substitution effect for the favored good dominates the income effect for the alternative good. However, with lexicographic ordering, no substitution effect is possible, and hence the change in relative demands is due only to income effects, constrained by the underlying lexicographic preferences.

The problem of lexicographic ordering has more than a theoretical interest. In the education area it has been suggested that school administrators may have lexicographically ordered objective functions. In particular, one behavioral model asserts that the first objective of educators is to reduce their pupil/teacher ratios to a generally agreed upon standard. Once that standard is obtained, other staff are acquired until that "standard" level is reached, after which the pupil/teacher ratio may be further reduced, providing sufficient resources are available. If

*Gerard Debreu, *Theory of Value*, John Wiley and Sons, New York, 1959.

lexicographic ordering significantly changes standard economic conclusions concerning the demand effects of relative price changes and if that type of ordering accurately describes a local educator's behavior, implications about the allocation of educational resources based on standard economic assumptions may be erroneous.

MAPPING LEXICOGRAPHICALLY ORDERED PREFERENCES

Although we assume that an individual's utility is a function of the quantities of the two goods, X_1 and X_2, he obtains, lexicographic preference ordering implies that within certain ranges, utility depends only on the amount of one of the goods. More specifically, we assume that the individual's preferences are such that until $\bar{X}_1$ of good X_1 is obtained, his utility depends on only the amount of X_1 he has available; further, additional amounts of X_1 beyond that quantity $\bar{X}_1$ do not add to his utility until he has obtained $\bar{X}_2$ of the good X_2. Thus we have three regions within the individual's utility map: region 1, where utility depends on quantities of X_1 from zero to $\bar{X}_1$; region 2, where increments in utility depend only on additional units of X_2 from zero to $\bar{X}_2$; and region 3, where increments in utility again depend on only increments of X_1 beyond $\bar{X}_1$, with at least $\bar{X}_2$ of the good X_2 available.

Figure 1 depicts this particular lexicographically ordered utility map. Region 1 encompasses the area below $\bar{X}_1$ along the entire range of X_2 (including zero X_2, i.e., the X_1 axis). With region 1, the utility map can be depicted by horizontal straight-line curves, with higher curves reflecting greater levels of utility (i.e., $U_2 > U_1$). Region 2 encompasses the area between the X_1 axis and the perpendicular to $\bar{X}_2$ above (but including) the perpendicular at $\bar{X}_1$.* All the utility curves in region 2 (with the exception noted in the footnote) are depicted by vertical straight-line curves. Since utility increases with X_2 in region 2, higher utility is indicated by curves lying to the right of the preceding one (i.e., $(U_4 > U_3 > U_2)$. The remaining area in the quadrant represents region 3, where utility curves are again shown by straight-line horizontals, with higher curves reflecting greater levels of utility $(U_6 > U_5 > U_4)$.

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*The perpendicular to $\bar{X}_1$ beyond $\bar{X}_2$ is also included in region 2. This reflects part of the highest utility curve in region 2. It is the one exception to the vertical straight-line utility curves contained in region 2.

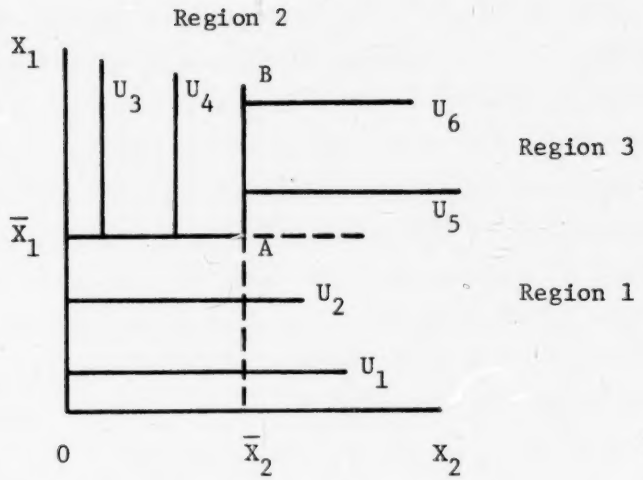

Fig. 1

The locus of points along the line $O\bar{X}_1AB$ is of special interest. Any point on this line represents the minimum combination of goods required to achieve that given utility level. If there are no "free" goods, then this locus represents the utility expansion path under utility maximizing behavior.* An important feature of this utility expansion path is that it is the same regardless of the relative prices for the two goods X_1 and X_2.

The constancy in the shape of the utility expansion path under lexicographic preference ordering helps explain the fundamental difference between this case and the standard economic preference function--the lack of any substitution effects. This is demonstrated in Fig. 2, which contains the utility expansion path from Fig. 1 and three budget restraints.

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*This is easily demonstrated by drawing a budget restraint and noting that maximum utility occurs where the budget restraint intersects the utility expansion path.

Restraint CD reflects the initial set of prices; restraints CE and FG, the assumed decrease in the price of X_2. Restraint FG represents both the "cost difference" and "compensating income" variations due to the price change. Because of the shape of the preference map, the tax required to constrain the consumer to his old bundle of goods or his old utility level at the new price set is exactly the same. More importantly, the change in prices net of the income effect (restraint FG) does not change the quantities of the goods purchased, even though the consumer is free to substitute along the restraint FG. In other words, a change in relative prices does not induce any substitution effect in favor of the lower priced good given lexicographically ordered preferences. With either budget restraint FG or CD, the consumer maximizes utility by purchasing $\bar{X}_1$ of X_1 and X_2^o of X_2.

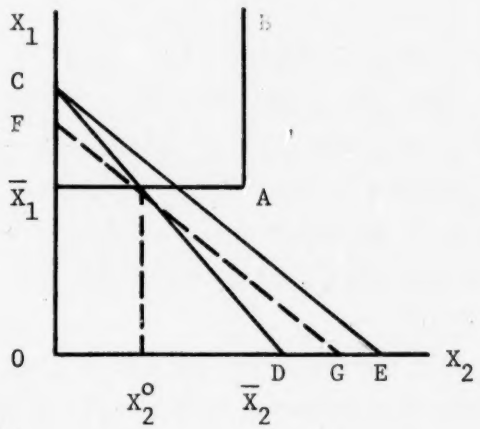

Fig. 2

CONSTRAINED INCOME EFFECTS

With standard assumptions concerning the utility function and two normal goods, the income effect derived from a fall in the price of one of the goods should produce increases in the demand for both goods. The change in the relative quantities obtained would depend upon the relative income elasticities for the two goods. With lexicographic ordering,

however, a positive income effect will produce an increase in demand for only one of the goods. The good undergoing an increase in its demand depends upon the region in which the impact of the price change is located. Basically, a price change favoring one good will produce an increased demand for that good only in those regions where utility depends upon the amount of that good consumed. For example, in region 2 utility levels depend upon the amount of X_2 consumed; hence a price change favoring X_2 which occurs in region 2 will yield an increase in the demand for X_2. However, a reduction in the price of X_2 that occurs in region 3 can produce no change in the demand for X_2 and an increase in the demand for X_1, even though both X_1 and X_2 are normal goods.*

IMPLICATIONS FOR ALLOCATION OF EDUCATIONAL RESOURCES

This model may have particular relevance in analyzing allocations of resources within public education for two reasons. First, the objectives of educators and the outputs of education are difficult to define and measure. Hence many analysts use measurable inputs as proxies for these objectives and/or outputs. Second, although there is no one dominant view, a prevalent notion concerning educators' objectives is that they contain inputs as arguments and that the arguments are lexicographically ordered. The statement that a school districts' primary objective is to achieve a target pupil/teacher ratio is symptomatic of this view. If this representation of educators' objectives is accurate, one's analysis of the impact of the current crises in the teacher labor market on the utilization of teachers and other educational personnel within local public schools could be significantly altered.

A traditional economic analysis of the resource allocation process among local public schools would suggest that the number of teachers (and indeed the staff mix between teachers and other educational personnel) ought to vary inversely with their relative costs. More specifically, as the costs of teachers decrease relative to other educational personnel

*If the change occurs in region 1, there is no impact at all; no X_2 is purchased, hence there is no income effect due to the savings from a price reduction.

(or even nonpersonnel inputs), local school districts would be expected to increase their demand for teachers relative to other educational personnel given maximizing behavior, normally ordered objectives, and production functions requiring alternative combinations of the two types of inputs. This expectation would prevail even if the inputs were arguments in the objective function.

If, however, educators have lexicographically ordered preferences, this expectation need no longer apply. Indeed, it is quite likely that relative price changes favorable for teachers, which produce a positive income effect, will result in an increase in the demand for other educational personnel while the demand for teachers is unchanged. This possibility is illustrated in Fig. 3, where the number of teachers per pupil and the number of other educational staff per pupil replaces X_1 and X_2 on the respective axes. As long as the district employs some other educational personnel and the impact of the price shift remains in region 2, the net effect of the change in relative prices favorable to teachers is to decrease the teachers/other staff mix from that depicted by ray Z_1, to that of ray Z_2.*

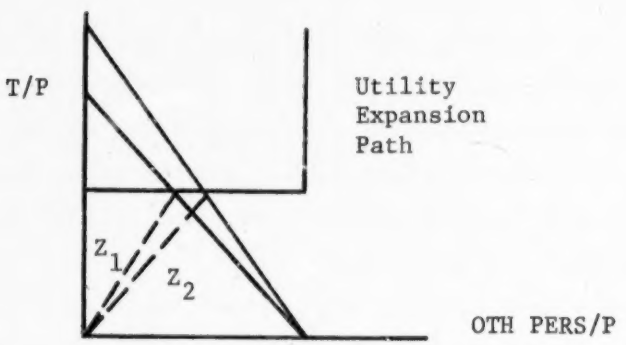

Fig. 3

*This assumes that the lexicographic ordering, more specifically the minimum target for T/P, is determined independently of relative factor prices and, more importantly, that the price change has a positive income effect. With a negative income effect a price change favorable to teachers would increase the teacher/other staff mix.

CONCLUSIONS

This paper has examined the effect of relative price changes on the demands for goods or factors of production which are lexicographically ordered in a utility or preference function. Because of this unique ordering, no substitution effects are possible. Moreover, the income effects are constrained and influence the demand for only one good or factor. Finally, it appears possible to observe a relative decrease in the demand for the "favored" good or factor even though both goods are normal or both factors have positive marginal products. If this model accurately reflects the preference ordering for local public school administrators, a standard economic analysis of the current crises in the teacher labor market could yield inappropriate conclusions regarding the adjustment process on the demand side.